IX. *On the Quantity and Velocity of the Solar Motion.* By William Herschel, *LL. D. F. R. S.*

Read February 27, 1806.

THE direction of the solar motion having been sufficiently ascertained in the first part of this Paper,* we shall now resume the subject, and proceed to an inquiry about its velocity.

The proper motions, when reduced to one direction, have been called quantities, to distinguish them from the velocities required in the moving stars to produce those motions. It will be necessary to keep up the same distinction with respect to the velocity of the solar motion; for till we are better acquainted with the parallax of the earth's orbit, we can only come to a knowledge of the extent of the arch which this motion would be seen to describe in a given time, when seen from a star of the first magnitude placed at right angles to the motion. There is, however, a considerable difference between the velocity of the solar motion and that of a star; for at a given distance, when the quantity of the solar motion is known its velocity will also be known, and every approximation towards a knowledge of the distance of a star of the first magnitude will be an approximation towards the knowledge of the real solar velocity; but with a star it will be otherwise; for though the situation of the plane in which it moves is

* Phil. Trans. for 1805, page 231.

given, the angle of the direction of its motion with the visual ray will still remain unknown.

As hitherto we have consulted only those proper motions which have a marked tendency to a parallactic centre, we ought now, when the question is to determine the velocity of the solar motion, to have in view the real motion of every star whose apparent motion we know; for as it would not be proper to assign a motion to the sun, either much greater or much less than any real motion which may be found to exist in some star or other, it follows that a general review of proper motions ought to be made before we can impartially fix on the solar velocity; but as trials with a number of stars would be attended with considerable inconvenience, I shall use only our former six in laying down the method that will be followed with all the rest.

Proportional Distance of the Stars.

We are now come to a point no less difficult than essential to be determined. Neither the parallactic nor real motion of a star can be ascertained till its relative distance is fixed upon. In attempting to do this it will not be satisfactory to divide the stars into a few magnitudes, and suppose *these* to represent the relative distances we require. There are not perhaps among all the stars of the heavens any two that are exactly at the same distance from us; much less can we admit that the stars which we call of the first magnitude are equally distant from the sun. And indeed, if the brightness of the stars is admitted as a criterion by which we are to arrange them, it is perfectly evident that all those of the first magnitude must differ as much in distance as they certainly do in lustre; yet imperfect as this may be, it is at present the only rule we have to go by.

The relative brightness of our six stars, may be expressed as follows: Sirius --- Arcturus - Capella -, Lyra -- Aldebaran . Procyon.

The notations here used are those which have been explained in my first Catalogue of the relative Brightness of the Stars;* but to denominate the magnitudes of these six stars so that they may with some probability represent the distances at which we should place them according to their relative brightness, I must introduce a more minute subdivision than has been commonly admitted, by using fractional distinctions, and propose the following arrangement.

Table VIII.

Proportional Distances of Stars.

Sirius - -	1,00	Lyra - -	1,30
Arcturus -	1,20	Aldebaran -	1,40
Capella - -	1,25	Procyon - -	1,40

The interval between Sirius and Arcturus is here made very considerable; but whoever will attentively compare together the lustre of these two stars, when they are at an equal altitude must allow that the difference in their brightness is fully sufficient to justify the above arrangement.

The order of the other four stars is partly a consequence of the distance at which Arcturus is placed, and of the comparative lustre of these stars such as it has been estimated by observations. But if it should hereafter appear that other

* Phil. Trans. for 1796, page 189.

more exact estimations ought to be subtituted for them, the method I have pursued will equally stand good with such alterations. I have tried all the known, and many new ways of measuring the comparative light of the stars, and though I have not yet found one that will give a satisfactory result, it may still be possible to discover some method of mensuration preferable to the foregoing estimations, which are only the result of repeated and accurate comparisons by the eye. Whenever we are furnished with more authentic data the calculations may then be repeated with improved accuracy.

Effect of the Increase and Decrease of the Solar Motion, and Conditions to be observed in the Investigation of its Quantity.

The following Table, in which the 2d, 4th, and 5th columns contain the sides of the parallactic triangle, is calculated with a view to show that an increase or decrease of the solar motion will have a contrary effect upon the required real motions of different stars; and as we are to regulate the solar velocity by these real motions, an attention to this circumstance will point out the stars which are to be selected for our purpose.

Table IX.

Stars and relative Distances.	Apparent Motion.	Solar Motion.	Parallactic Motion.	Real Motion.	Velocities.
Sirius 1,00	1″,11528	1,0 1,5 2,0	0″,67768 1,01652 1,35536	+ 0″,46518 + 0,21701 − 0,32776	465175 217007 327755
Arcturus 1,20	2″,08718	1,0 1,5 2,0	0,53579 0,80368 1,07158	+ 1,57389 + 1,30478 + 1,01561	1888670 1565735 1218736
Capella 1,25	0″,46374	1,0 1,5 2,0	0,79593 1,19390 1,59186	− 0,42159 − 0,79637 − 1,18662	526987 995465 1483270
Lyra 1,30	0″,32435	1,0 1,5 2,0	0,32542 0,48812 0,65083	− 0,47065 − 0,59923 − 0,74135	611839 778995 963750
Aldebaran 1,40	0″,12341	1,0 1,5 2,0	0,65117 0,97676 1,30234	− 0,53208 − 0,85737 − 1,18283	744913 1200324 1655967
Procyon 1,40	1″,23941	1,0 1,5 2,0	0,66394 0,99591 1,32788	+ 0,59548 + 0,30731 − 0,23385	833665 430227 327390

The real motion of Arcturus contained in the 5th column compared with that of Aldebaran, shows that when the solar motion is increased from 1,0 to 1,5 and to 2″,0 the real motion of Arcturus will be gradually diminished from 1,57 to 1,30 and to 1″,02, while that of Aldebaran undergoes a contrary change from 0,53 to 0,86 and to 1″,18. We may also notice that Capella and Aldebaran, which have a negative sign prefixed to their real motions when the solar motion is 1″,0 are affected differently from Arcturus, Sirius, and Procyon, which have a positive sign; and that even the motions of the two last become negative when the solar motion is increased beyond a certain point. It may be easily understood that the motion of Arcturus itself would become negative were we to increase the solar motion till the parallactic motion of this star should exceed its apparent motion.

From these considerations it appears, that a certain equalization, or approach to equality may be obtained between the motions of the stars, or between that of the sun and any one of them selected for the purpose; for instance, the motions of Arcturus and Aldebaran being contrary to each other, may be made perfectly equal by supposing the sun's annual motion to be 1″,85925. For then we shall have the real annual motion of Arcturus towards the parallactic centre 1″,091, and that of Aldebaran towards the opposite part of the heavens, in which the solar apex is placed, will be 1″,091 likewise; the first in a direction 55° 29′ 39″ south-preceding, the latter 88° 16′ 31″ north-following their respective parallels; and a composition of these motions with the parallactic ones arising from the given solar motion, will produce the apparent motions of these stars which have been established by observation. But since Arcturus, by the hypothesis which has been adopted in Table VIII. is a nearer star than Aldebaran, the velocities of the real motions, describing these equal arches will be 1309109 in the former and 1527780 in the latter. And it is not the arches but these velocities that must be equalized. Therefore, in order to have this required equality, let the solar motion be 1″,718865 then will a velocity of 1399478 in Arcturus, and 1399842 in Aldebaran, which are sufficiently equal, occasion such angular real motions in the two stars as will bring them, when compounded with their parallactic motions, to the apparent places in which we find them by observation.

Before we proceed, it will be proper to obviate a remark that may be made against this way of equalization or approach to equality. We have said that the calculated velocities are such as would be true if the stars were at the assumed distances, and if their real motions were performed in lines at right angles to the visual ray; to which it may here be objected that the last of these assumptions is so far from having any proof in its favour that even the highest probability is against it. We may admit the truth of what the objection states, without apprehending that any error could arise on that account, if the solar motion were determined by this method. For if the stars do not move at right angles to the visual ray, their real velocity will exceed the calculated one; so that in the first place we should certainly have the minimum of their velocities: and if we were obliged, for want of data to leave the other limit of the motion unascertained, it must be allowed to be a considerable point gained if we could show what is likely to be the least velocity of the solar motion; but a more satisfactory defence of the method is, that if we were to assume a mean of all the angular deviations from the perpendicular to the visual ray that may take place in the directions of the real motions of the stars, the only position we could fix upon as a mean would be an inclination of 45 degrees. For in this case the chance of a greater or smaller deviation would be equal; and when a number of stars are taken, the deviations either way might then be supposed to compensate each other; but what is chiefly to our purpose, not only the angle of 45 degrees, but also any other, that might be fixed upon as a proper one to represent the mean quantity of sidereal motions, would lead exactly to the same result of the solar velocity to be investigated. For if the velocities of any two stars were equalized, when their motions are supposed to be perpendicular to the visual ray, they would be as much so when they make any other given angle with it; and it is the equalization or approach to equality and not the quantity of the velocities that is the spirit of this method. I have only to add, that an equalization of the solar motion with that of any star selected for the purpose may be had by a direct method of calculation, and will therefore be of great use in settling the rate of the motion to be determined.

It must be evident from what has been said, that a certain mean rate, or middle rank, should be assigned to the motion of the sun, unless very sufficient reasons should induce us to depart from this condition. To obtain this end must consequently be our principal aim; and if we can at the same time bring the sidereal motions to a greater equality among each other, it will certainly be a very proper secondary consideration.

There are two ways of taking a mean of the sidereal motions, one of them may be called the rate and the other the rank. For instance, a number equal to the mean rate of the six numbers, 2, 6, 13, 15, 17, 19, would be 12; but one that should hold a middle rank between the three highest and three lowest of the six would be 14. In assigning the rate of the solar motion it appears to be most eligible that it should hold a middle rank among the sidereal velocities. We shall however find that nearly the same result will be obtained from either of the methods.

With respect to our second consideration, we may see that it also admits of a certain modification by the choice of the solar motion; for in Table IX. when this motion is 1″,5 the velocity of Arcturus 1565735, will exceed that of Sirius, 217007, more than seven times; whereas a solar motion of 1″ will give us the proportional velocities of these stars as 188867 to 465174; and the former will then exceed the latter only four times.

Calculations for drawing Figures that will represent the observed Motions of the Stars.

The necessary calculations for investigating the solar motion are of considerable extent, and may be divided into two classes, the first of which will remain unaltered whatsoever be the solar motion under examination, while the other must be adjusted to every change that may be required.

The direction of the sun remaining as it has been settled in the first part of this Paper, the permanent computation of each star will contain the annual quantity of the observed or apparent motion, its direction with the parallel of the star, its direction with the parallactic motion, and its velocity. The changeable part will consist of the angular quantity of the real motion, the parallactic direction of this motion, and its velocity.

Before we can make a calculation of the required velocities, we must fix upon the probable relative distance of the rest of the stars, in the same manner as we have done with the first six. In this I have thought it advisable to distinguish the stars that, from their lustre, may be called principal, and have limited their extent to the brightest of the second magnitude, on account of the uncertainty which still remains about their progressive distances. For though it appears reasonable to allow that the bright stars of the second magnitude may be twice as far from us as those of the first, it will admit of some doubt whether this rule ought to be strictly followed up to the 3d, 4th, 5th, and 6th magnitude; especially when it is not easy to ascertain the boundaries which should limit the magnitudes of very small stars.

The number of these principal stars is 24. The remaining 12 are also arranged by admitting that their magnitudes express their relative distances; and notwithstanding the doubtfulness we have noticed, their testimony with respect to the proper quantity of a solar motion, though it should be received with some diffidence, must not be neglected; some considerable alteration in their supposed distances, however, would have but little effect upon the conclusions intended to be drawn from their velocities.

The following Table contains the result of the calculations that relate to the permanent quantities. In the first and second columns, we have the names of the stars, and their assigned relative distances. The third gives the apparent angular motions, and the fourth their direction. The fifth contains the direction of the same motions, with respect to the parallactic motions arising from the given solar direction; and the sixth gives the velocity of the stars which produce the quantity of the apparent motions.

Table X.

Names of the Stars.	Proportional Distances.	Apparent Motions.	Direction with the Parallel.	Direction with the parallactic Motion.	Velocity of the Stars.
Sirius -	1,00	1″,11528	68.49.40,7 *sp*	10.24.44,3 *sf*	1115281
Arcturus	1,20	2,08718	55.29.42,0 *sp*	0. 0. 3 *sp*	2504621
Capella -	1,25	0,46374	71.35.22,4 *sf*	24.40.21 *sf*	579668
Lyra -	1,30	0,32435	56.20.57,3 *nf*	92.49.30 *nf*	421657
Rigel -	1,35	0,16273	79.29.33,9 *np*	159.28. 1 *np*	219684
α Orionis	1,35	0,13038	85.38.14,6 *nf*	169.18.58 *np*	176010
Procyon	1,40	1,23941	50. 2.24,5 *sp*	9.40.46 *sp*	1735172
Aldebaran	1,40	0,12341	76.29.37,3 *sf*	13.41.48 *sf*	172778
Pollux -	1,42	0,65037	0. 0. 0 *prec.*	61.30.34 *sp*	923523
Spica -	1,44	0,19102	84. 5. 1,8 *np*	144.13.16 *np*	275065
Antares -	1,46	0,26000	90. 0. 0 *north*	178.57.44 *np*	379600
Altair -	1,47	0,71912	48.40.12,0 *nf*	103.17.29 *nf*	1057105
Regulus -	1,48	0,22886	20.27.37,5 *np*	70. 9.20 *sp*	338711
β Leonis	1,50	0,55324	7.16. 8,4 *sp*	40.34.31 *sp*	829856
β Tauri -	1,50	0,10039	84.58.27,1 *sf*	13.17.11 *sf*	150579
Fomalhaut	1,50	0,30698	11,16.16,3 *nf*	16.47. 5 *sf*	460469
α Cygni	1,60	0,06440	27.45.56,3 *np*	177.31.39 *np*	103036
Castor -	2,00	0,13294	17.30.40,6 *sp*	45.25.43 *sp*	265869
α Ophiuchi	2,00	0,07698	40.30.24,8 *sf*	33.29.28 *sf*	153955
α Coronæ	2,00	0,23279	7.24.15,4 *sf*	105. 0.43 *nf*	465587
α Aquarii	2,00	0,20615	67.10.17,1 *np*	162.43.46 *nf*	412295
α Andromedæ	2,00	0,09268	40.20.48,2 *sf*	12.55.11 *sf*	185360
α Serpentis	2,00	0,21913	60. 7.12,5 *nf*	161.34. 4 *nf*	438257
α Pegasi	2,00	0,18917	72. 5.16,0 *np*	157.45.25 *nf*	378338
α Hydræ	2,30	0,16598	57.30.24,8 *np*	107. 6.24 *np*	381763
α² Libræ -	2,40	0,18376	54.42.52,9 *np*	127. 3. 7 *np*	441022
γ Pegasi	2,50	0,17355	59.48. 7,9 *np*	174. 5.15 *nf*	433880
α Arietis -	2,50	0,11587	37. 9.15,9 *sf*	29.32.47 *sf*	289685
α Ceti -	2,80	0,14406	33.44. 2,9 *np*	141.18.55 *np*	403356
α Herculis	3,00	0,23000	90. 0. 0 *north*	168.23.41 *nf*	690000
β Virginis	3,00	0,77706	17.59.25,5 *sf*	111.11.44 *nf*	2331169
γ Aquilæ	3,00	0,19320	55.54.41,7 *np*	178.25.20 *nf*	579589
α² Capricorni	3,50	0,26452	79.23.35,3 *nf*	136.21.18 *nf*	925819
β Aquilæ	4,00	0,35127	85. 7.37,0 *sp*	39.49.15 *sp*	1405079
α′ Capricorni	4,20	0,28000	90. 0. 0 *north*	146.59.44 *nf*	1176000
α′ Libræ	6,00	0,20898	59.27.58,4 *np*	131.46. 7 *np*	1253875

The contents of this Table will enable us to examine the motions of the stars in different points of view. For instance, by the apparent motions in the third column, and their directions in the fourth, a figure may be drawn which will represent the actual state of the heavens, with respect to those annual changes in the situations of our 36 stars, which in astronomical tables are called their proper motions.

Fig. 1, Plate IV. gives us these motions brought into one view, so that by supposing successively every one of the stars to be represented by the central point of the figure, we may see the angular quantity and direction of the several annual proper motions represented by the line which is drawn from the centre to each star. By this means we have the comparative arrangement and quantity of these movements with respect to their directions.

Fig. 3 represents the same motions, but instead of being drawn so as to show their directions with regard to the several meridians and parallels of the stars, they are laid down by the angles contained in the fifth column; and will therefore indicate their arrangement with respect to a line drawn from the solar apex towards the parallactic centre. These directions will remain the same, whatever may be the velocity of the solar motion upon which we shall ultimately fix, provided no change be made in the situation of the apex towards which the sun has been admitted to move.

In these two figures, the lines drawn from the centre give us only the angular changes of the places that have been either observed or calculated, and not the velocities which are required in the stars to produce them. It will therefore be necessary to represent the velocities by two other figures, in which the same directions are preserved, but where the extent of each line is made proportional to the distance of the stars in the second column.

Fig. 2 is drawn according to this plan; the angles of the directions remain as in the fourth column, but the lines are lengthened so as to give us the velocities contained in the sixth.

In Fig. 4, the angles of the 3d figure are preserved, but the lines are again lengthened as in Fig. 2.

N. B. These two last figures would have been of an inconvenient size if they had been drawn on the same scale with the two foregoing ones, for which reason, in comparing the 2d and 4th with the 1st and 3d, it must be remembered that the former are reduced to one half of the dimensions of the latter.

Remarks on the sidereal Motions as they are represented from Observation.

As we have now before us a set of figures which give a complete view of the result of the calculations contained in the Xth Table, we may examine the arrangements of the stars, and draw a few conclusions from them, that will throw some light upon the subject of our present inquiry.

In the first place, then, we have to observe in Fig. 1, that 17 out of the 21 stars, whose motions are directed towards the north, are crowded together into a compass of little more than $76\frac{1}{2}$ degrees. But this figure, as we have shown, is drawn from observation. We are consequently obliged to conclude, that, if these motions are the real ones, there must be some physical cause which gives a bias to the directions in which the stars are moving; if so, it would not be improbable that the sun, being situated among this group of stars, should partake of a motion towards the same part of the heavens.

Our next remark concerns the velocity of the sidereal motions; and therefore we must have recourse to Fig. 2, where we perceive that the greatest motions are not confined to the brightest stars. For instance, the velocity of β Virginis is but little inferior to that of Arcturus, and exceeds the velocity of Procyon. Likewise the velocities of β Aquilæ, α′ Libræ, and α′ Capricorni, surpass that of Sirius; and an inspection of the rest of the figure will be sufficient to show how very far the velocities of Capella, Lyra, Rigel, α Orionis, Aldebaran, and Spica, are exceeded by those of many other stars.

If we look at the arrangement of the stars with respect to the direction of the solar motion, we find in Fig. 3, that a somewhat different scattering of them has taken place; but still most of the stars appear to be affected by some cause which tends to lead them to the same part of the heavens, towards which the sun is moving; and the directions of the greatest number of them are not very distant from the line of the solar motion.

The whole appearance of this figure presents us with the idea of a great compression above the centre, arising from some general cause, and a still greater expansion in the lower part of it. The considerable projection of a few stars on both sides, is however a plain indication that the compressing or dilating cause does not act in their directions.

When the velocity of the stars, represented in the same point of view in Fig. 4, is examined, we find a particularity in the direction and comparative velocities in the largest stars that must not be overlooked. Four of them, Rigel, α Orionis, Spica, and Antares, have a motion towards that part of the heavens in which the solar apex is placed, and their motions are very slow. Three other stars of the 1st magnitude, Arcturus, Procyon, and Sirius, move towards the opposite part of the heavens, and their motions, on the contrary, are very quick.

The direction of the motion of Aldebaran, compared with its small velocity, is no less remarkable; and seems to be contrary to what has been pointed out with the three last mentioned stars; we shall however soon have an opportunity of showing that it is perfectly consistent with the principles of the solar motion.

The Solar Motion and its Direction assigned in the first Part of this Paper are confirmed by the Phenomena attending the observed Motions of the 36 Stars.

An application of some of the foregoing remarks will be our next subject; and I believe it will be found, that in the first place they point out the expediency of a solar motion. That next to this they also direct us to the situation of the apex of this motion: and lastly, that they will assist us in finding out the quantity requisite for giving us the most satisfactory explanation of the phenomena of the observed proper motions of the stars.

In examining the second figure, it has been shown that no less than six stars of the first magnitude, namely, Capella, Lyra, Rigel, α Orionis, Aldebaran, and Spica, have less velocity than nine or ten much smaller stars. Aldebaran and α Orionis indeed have so little motion that there are but three stars in all the 36 that have less. But the situation of these bright stars, from their nearness, must be favourable to our perceiving their real motions if they had any, unless they were counteracted by some general cause that might render them less conspicuous. Now to suppose that the largest stars should really have the smallest motions, is too singular an opinion to be maintained; it follows, therefore, that the apparently small motions of these large stars is owing to some general cause, which renders at least some part of their real motion invisible to us. But when a solar motion is introduced, the parallax arising from that cause will completely account for the singularity of these slow motions.

If the foregoing argument proves the expediency of a solar motion, its direction is no less evidently pointed out by it. For if the parallax occasioned by the motion of the sun is to explain the appearances that have been remarked, it will follow, that a direction in opposition to the motion of Arcturus, will answer that end in the most satisfactory manner. That compression, for instance, which has been remarked in the motions of the stars moving toward the solar apex in Fig. 3, and which is so completely accounted for by a parallactic motion arising from the motion of the sun, points out the direction in which the sun should move, in order to produce this required parallactic motion. The expansion of the motions that are in opposition to the former is evidently owing to the same parallactic motions, which in this direction unite with the real motions of the stars; and as, in the former case, the observed motions are the differences between the parallactic and real motions, so here they are the sum of them.

The remark that stars having a side motion, are not affected by the cause of the compression or expansion, which acts upon the rest, is perfectly explained; for a parallactic motion, in the direction of the motion of Arcturus, can have no effect in lengthening or shortening the perpendicular distance to which a star may move in a side direction.

I have only to add, that the small velocities of Rigel, α Orionis, Spica, and Antares, in Fig. 4, as well as the great velocities of Arcturus, Procyon, and Sirius, point out the same apex which in the first part of this Paper has already been established by more extended computations.

The case of Aldebaran, though seemingly contrary to what has been shown, confirms the same conclusions. This will appear by considering that a star, moving towards the solar apex with a greater real motion than its parallactic one, must continue apparently to move in its real direction; but should a star, such as Aldebaran, move towards the apex with less velocity than the parallactic motion which opposes it, there will arise a change of direction, and the star will be seen moving towards the opposite part of the heavens.

Trial of the Method to obtain the Quantity of the Solar Motion by its Rank among the sidereal Velocities.

According to the conditions that have been explained, a calculation may be made with a view of equalizing the velocities of the sun and the star α Orionis; and the result of it will show that the proposed equality will be obtained when the solar motion is 1″,266230. It will moreover be found than so small an increase of this motion as 0″,01 would give us 19 stars with less, and 17 with more velocity than that which the calculation assigns to the sun; this consequently fixes one of the limits to which the solar motion ought not to come up, if we intend it should hold a middle rank among the sidereal velocities.

On the other hand, by a similar calculation of the velocities of the star Pollux and the sun, it appears that a solar motion of 0″,967754 will make them equal; and that a diminution of this motion not exceeding 0″,01 would give us 19 stars moving at a greater rate than the sun, and only 17 falling short of its velocity. This consequently fixes the other limit to which the solar motion ought not to be depressed. And thus it appears by this method, that the quantity we are desirous of ascertaining, is confined within very narrow bounds, and that by fixing upon a mean of the two limits, we may have the rank of the solar motion true to less than 0″,15.

Calculations for investigating the Consequences arising from any proposed Quantity of Solar Motion, and for delineating them by proper Figures.

Before we can justly examine the real motions of stars which it will be necessary to admit in consequence of a given solar motion, it will be convenient to have them represented in two figures that we may see their arrangement and extent; and as a calculation of the required particulars will oblige us to fix upon a certain quantity, we shall take the motion that has been ascertained to belong to the middle rank of the sidereal velocities for a pattern. The result of the necessary calculations is as follows.

Table XI.

Names.	Parallactic Motion.	Real Motion.	Parallactic Angle.	Velocity.
	"	"	° ′ ″	
Sun - -	0,00000	1,116992	00.00.00	1116992
Sirius - -	0,75697	0,395212	149.20. 6 *sf*	395212
Arcturus -	0,59847	1,488713	179.59.55,7 *sp*	1786455
Capella -	0,88905	0,506123	22.29.12,5 *nf*	632654
Lyra - -	0,36349	0,498949	40.29.14 *nf*	648634
Rigel - -	0,55470	0,709381	4.36.52 *np*	957665
α Orionis -	0,71410	0,842559	1.38.38 *np*	1137455
Procyon -	0,74161	0,523428	156.32.21 *sp*	732799
Aldebaran -	0,72736	0,608148	2.45.15 *nf*	851407
Pollux - -	0,78643	0,743971	50.12.11 *np*	1056439
Spica - -	0,74009	0,902004	7. 6.44 *np*	1298886
Antares -	0,74110	1,000835	0.16.10,5 *np*	1461219
Altair - -	0,64544	1,071042	40.48. 4 *nf*	1574431
Regulus -	0,75095	0,706833	17.43.53 *np*	1046113
β Leonis -	0,68003	0,443842	54.10.14,5 *np*	665763
β Tauri -	0,73063	0,633317	2. 5.15,5 *nf*	949976
Fomalhaut	0,66693	0,383414	13.22. 5,5 *nf*	575121
α Cygni -	0,46516	0,529503	0.18. 2,2 *np*	847204
Castor - -	0,55841	0,474647	11.30.32 *np*	949293
α Ophiuchi	0,35202	0,290934	8.23.43 *nf*	581869
α Coronæ -	0,23427	0,370580	37.21.17 *nf*	741160
α Aquarii -	0,55743	0,756754	4.38.19,5 *nf*	1513508
α Andromedæ	0,55389	0,464035	2.33.34 *nf*	928071
α Serpentis	0,38655	0,598458	6.38.54 *nf*	1196917
α Pegasi -	0,55567	0,734265	5.35.47,5 *nf*	1468530
α Hydræ -	0,46554	0,538281	17. 8.26 *np*	1238646
α^{2} Libræ -	0,43377	0,563892	15. 4.29 *np*	1353342
γ Pegasi -	0,44540	0,618272	1.39.27 *nf*	1545679
α Arietis -	0,43893	0,342934	9.35.29,5 *nf*	857336
α Ceti -	0,33271	0,454165	11.26. 5,5 *np*	1271662
α Herculis	0,21909	0,446795	5.56.38,5 *nf*	1340388
β Virginis -	0,36039	0,967572	48.29. 2,5 *nf*	2902716
γ Aquilæ -	0,30898	0,502168	0.36.25 *nf*	1506503
α^{2} Capricorni	0,31390	0,537285	19.51.52,5 *nf*	1880497
β Aquilæ -	0,24370	0,226458	96.36.59,5 *sp*	905830
α′ Capricorni	0,26151	0,519230	17. 4.54,5 *nf*	2180769
α′ Libræ -	0,17347	0,349371	26.29.44,5 *np*	2096229

By the contents of this Table, Fig. 5 is drawn with the lines contained in the third column and the angles of the fourth; the scale of it is that of the 5th and 3d figures; and it represents the directions and angular quantities of the real motions that are required to compound with the parallactic effects of the second column, so as to produce those annual proper motions which are established by observation.

Fig. 6 is drawn on the reduced scale of the 2d and 4th figures. The lines make the same angles with the direction of the solar motion as before, but their lengths are in the proportion of the velocities contained in the last column.

Remarks that lead to a necessary Examination of the Cause of the sidereal Motions.

The first particular that will strike us when we cast our eye on Fig. 5, is the uncommon arrangement of the stars. It seems to be a most unaccountable circumstance that their real motions should be as represented in that figure; indeed, if we except only ten of the stars, all the rest appear to be actuated by the same influence, and, like faithful companions of the sun, to join in directing their motions towards a similarly situated part of the heavens.

This singularity is too marked not to deserve an examination; for unless a cause for such peculiar directions can be shown to exist, I do not see how we can reconcile them with a certain equal distribution of situations, quantities, and motions, which our present investigation seems to demand. In order to penetrate as far as we can into this intricate subject, we shall take a general view of the causes of the motions of celestial bodies.

A motion of the stars may arise either from their mutual gravitation towards each other, or from an original projectile force impressed upon them. These two causes are known to act on all the bodies belonging to the solar system, and we may therefore reasonably admit them to exert their influence likewise on the stars. But it will not be sufficient to know a general cause for their motions, unless we can show that its influence will tend to make them go towards a certain part of the heavens rather than to any other. Let us examine how these causes are acting in the solar system.

The projectile motions of the planets, the asteroids, and the satellites, excepting those of the Georgium Sidus, are all decidedly in favour of a marked singularity of direction. We may add to them the comet of the year 1682, whose regular periodical return in 1759 has sufficiently proved its permanent connection with the solar system. Here then we have not less than 23 various bodies belonging to the solar system to show that this cause not only can, but in the only case of which we have a complete knowledge, actually does influence the celestial motions, so as to give them a very particular appropriate direction. Even the exception of the Georgian satellites may be brought in confirmation of the same peculiarity; for though they do not unite with the rest of the bodies of our system, they still conform among each other to establish the same tendency of a similar direction in their motion round the primary planet. And thus it is proved that the similar direction of the motion of a group of stars may be ascribed to their similar projectile motions without incurring the censure of improbability.

Let us however pursue the objection a little farther, and as we have shown that the celestial bodies of the solar system actually have these similar projectile motions, it may be required that we should also prove that the stars have them likewise; since the appearances in Fig. 5 may otherwise be looked upon as merely the consequence of the assumed solar motion. To this I answer, that setting aside the solar motion, and allowing the observations of astronomers on the proper motion of the stars to give us the real direction and angular quantity of these motions, even then the same similarity will equally remain to be accounted for. In my examination of Fig. 1 and 3, it has been shown that we ought to ascribe the similar directions of the sidereal motions to some physical cause, which probably exerts its influence also on the solar motion; therefore in reverting to those figures I may be said to appeal to the actual state of the heavens, for a proof of what has been advanced, with respect to the similarity of the directions of projectile motions.

Having thus examined one cause of the sidereal motions, and shown that as far as we are acquainted with its mode of acting in the solar system, it is favourable to a similarity of direction; and that moreover, if we ascribe the motion of the stars to it, we have also good reason, from observation, to believe it to be in favour of the same similarity; we may in the next place proceed to consider the mutual gravitation of the stars towards each other. This is an acknowledged principle of motion, and the laws of its exertion being perfectly known, we shall in this inquiry meet with no difficulty relating to its direction, which is always towards the attracting body.

Considerations of the attractive Power required for a sufficient Velocity of the sidereal Motions.

As attraction is a power that acts at all distances, we ought to begin by examining whether the motions of our stars can be accounted for by the mutual gravitation of neighbouring stars towards each other, or by a periodical binal revolution of them about a common centre of gravity; or whether we ought not rather to have recourse to some very distant attractive centre. This may be decided by a calculation of the effects arising from the laws according to which the principle of attraction is known to act. For instance, let the sun and Sirius be two equal bodies placed in the most favourable situation to permit a mutual approach by attraction: that is, let them be without projectile motions, and removed from all other stars which might impede their progress towards each other, by opposite attractions. Then, by calculation, the space over which one of them would move in a year, were the matter of both collected in the other as an attractive centre, would be less than a five thousand millionth part of a second; supposing that motion to be seen by an eye at the distance of Sirius, and admitting the parallax of the whole orbit of the earth on this star to be one second.

This proves evidently that the mere attraction of neighbouring stars acting upon each other cannot be the cause of the sidereal motions that have been observed.

In the case of supposed periodical binal revolutions of stars about a common centre of gravity, where consequently projectile motions must be admitted, the united power of the connected stars, provided the mass of either of them did not greatly exceed that of the sun, would fall very short of the attraction required to give a sufficient velocity to their motions. The star Arcturus for example, which happens to move, as is required, in an opposite direction to the proposed solar motion, were it connected with the sun, and had the proper degree of necessary projectile motion, could not describe an arch of 1″ of its orbit, about their common centre, in less than 102 years; and though the opposite motion of the sun, by a parallactic effect would double that quantity, it still would fall short of the change we observe in this star in the course of a single year.

Other considerations are still more against the admission of such partial connections: they would intirely oppose the similarity of the directions of the sidereal motions that have been proved to exist, and which we are now endeavouring to explain.

Let us then examine in what manner a distant centre of attraction may be the cause of the required motions. By admitting this centre to be at a great distance, we shall have its influence extended over a space that will take in a whole group of stars, and thus the similar directions of their motions will be accounted for. Their velocities also may be ascribed to the energy of the centre, which may be sufficiently great for all the purposes of the required motions. A circumstance, however, attends the directions of the motions to be explained, which shows that a distant centre of attraction alone will not be sufficient; for these motions, as we may see in Fig. 3, though pretty similar in their directions, still are diverging; whereas if they were solely caused by attraction, they would converge towards the attracting centre, and point out its situation. It is therefore evident that projectile motions must be combined with attraction, and that the motions of the stars when regulated in this manner, are not unlike the disposition by which the bodies of the solar system are governed. If we pursue this arrangement, it will be proper to consider the condition, and probable existence of such a centre of attraction.

There are two ways in which a centre of attraction, so powerful as the present occasion would require, may be constructed; the most simple of them would be a single body of great magnitude; this may exist, though we should not be able to perceive it by any superiority of lustre; for notwithstanding it might have the usual starry brightness, the decrease of its light arising from its great distance would hardly be compensated by the size of its diameter; but to have recourse to an invisible centre, or at least to one that cannot be distinguished from a star, would be intirely hypothetical, and, as such, cannot be admitted in a discussion, the avowed object of which is to prove its existence.

The second way of the construction of a very powerful centre, may be joint attraction of a great number of stars united into one condensed group.

The actual existence of such groups of stars has already been proved by observations made with my large instruments; many of those objects, which were looked upon as nebulous patches, having been completely resolved into stars by my 40 and 20-feet telescopes. For instance, the nebula discovered by Dr. Halley in the year 1714, in which the discoverer, and other observers after him, have seen no star, I have ascertained to be a globular cluster, containing, by a rough calculation, probably not less than fourteen thousand stars. From the known laws of gravitation, we are assured that this cluster must have a very powerful attractive centre of gravity, which may be able to keep many far distant celestial bodies in control.

But the composition of an attractive centre is not limited to one such cluster. An union of many of them will form a still more powerful centre of gravitation, whose influence may extend to a whole region of scattered stars. To prove that I argue intirely from observations, I shall mention that another nebula, discovered by Mr. MESSIER in the year 1781, is, by the same instruments, also proved to consist of stars; and though they are seemingly compressed into a much smaller space, and have also the appearance of smaller stars, we may fairly presume that these circumstances are only indications of a greater distance, and that, being a globular cluster, perfectly resembling the former, the distance being allowed for, it is probably not less rich in the number of its component stars. The distance of these two clusters from each other is less than 12 degrees, and we are certain that somewhere in the line joining these two groups there must be a centre of gravitation, far superior in energy to the single power of attraction that can be lodged in either of the clusters.

I have selected these two remarkable objects merely for their sinuation, which is very near the line of the direction of the solar motion; but were it necessary to bring farther proof of the existence of combined attractions, the numerous objects of which I have given catalogues* would amply furnish me with arguments.

If a still more powerful but more diffused exertion of attraction should be required than what may be found in the union of clusters, we have hundreds of thousands of stars, not to say millions, contained in very compressed parts of the milky way, some of which have already been pointed out in a former Paper.† Many of these immense regions may well occasion the sidereal motions we are required to account for; and a similarity in the direction of these motions will want no illustration.

With regard to the situation of the condensed parts of the milky way, and of the two clusters that have been mentioned, we must remark, that the seat of attraction may be in any part of the heavens whatsoever; for when projectile motions are given to bodies that are retained by an attractive centre, they may have any direction, even that at right angles to its situation not excepted.

It will give additional force to the arguments I have used for the admission of far distant centres of attraction, as well as projectile motions in the stars that are connected with them, when we take notice that, independent of the solar motion, and setting that intirely aside, the action of these causes will be equally required to explain the acknowledged proper motions of the stars. For if the sun be at rest, then Arcturus must actually change its place more than 2″ a year, and consequently this and many other stars, which are well known to change their situation, must be supposed to have

* Phil. Trans. for 1786, page 457; for 1789, page 212; for 1802, page 477.

† Ibid. for 1802, page 495.

projectile motions, and to be subject to the attraction of far distant centres.

Determination of the Quantity of the Solar Motion.

If I am not mistaken, it will now be allowed that no objection can arise against any solar velocity we may fix upon, for want of a cause that may be assigned to act upon the sun, and many stars, so as to account for their motions, and similar tendency towards a certain part of the heavens; we may consequently proceed in examining whether the quantity that has been assumed for calculating the contents of the XIth Table, will sufficiently come up to the conditions we have adopted for directing our determination.

In Fig. 6 we have the velocities of the 36 stars delineated, and by examining the last column of the Table from which they are taken, we find that the parallactic effects arising from the proposed solar motion require the velocity of 18 stars to exceed that of the sun, and exactly the same number to be inferior to it; so far then the rank which has been assigned to the solar motion is a perfect medium among the sidereal velocities.

If we examine in the next place how this motion will agree with a mean rate deduced from the velocities in the above mentioned column, we find a 36th part of their sum to be 1196550. A solar motion, therefore, which agrees with this mean rate will differ from one assigned by the middle rank no more than 0″,079558; and, on account of the smallness of this quantity, the calculations required to lessen it, by some little increase of the solar motion, might well be dispensed with; but if we were desirous of greater precision, the secondary purpose, next to be considered, would rather incline us to an opposite alteration.

The great disparity of the sidereal motions, which has been mentioned as an incongruity in the first part of this Paper, and has more evidently been shown to exist when we examined the representations of these motions in the 3d figure, is the next point we have to consider in the effect of the solar motion. Let us see how far we have been successful in lessening the ratio these velocities bear to each other. The last column of the Xth Table contains them as they must have been admitted if the sun had been at rest. The proportion of the quickest motion to the slowest is there as 2504621 to 103036; and the velocity of one is therefore above 24 times greater than that of the other. But in consequence of the solar motion we have used, the two extreme velocities are reduced to 2902716 and 395212; which gives a proportion of less than $7\frac{1}{2}$ to 1.

If the quantity of the solar motion were lessened to 1″, we might bring the ratio of the extreme velocities so low as 6 to 1; but as the middle rank has already given it a little below the mean rate, I do not think that we ought to lower it still more; so that when all circumstances are properly considered, there is a great probability that the quantity assumed in the last calculation may not be far from the truth. It appears, therefore, that in the present state of our knowledge of the observed proper motions of the stars, we have sufficient reason to fix upon the quantity of the solar motion to be such as by an eye placed at right angles to its direction, and at the distance of Sirius from us, would be seen to describe annually an arch of 1″,116992 of a degree; and its velocity, till we are acquainted with the real distance of this star, can therefore only be expressed by the proportional number of 1116992.

Concluding Remarks and Inferences.

We have now only to notice a few remarks that may be made, by way of objection to the solar motion I have fixed upon. If the quantity of this motion is to be assigned by the mean rank of sidereal velocities, it may be asked, will not the addition of every star, whose proper motion shall be ascertained, destroy that middle rank, which has been established? To this I shall answer, that future observations may certainly afford us more extensive information on the subject, and even show that the solar motion should not exactly hold that middle rank, which from various motives we have been induced to assign to it; but at present it appears, that according to the doctrine of chances, a middle rank among the sidereal velocities must be the fairest choice, and will remain so, unless, what is now a secondary consideration, should hereafter become of more importance than the first. That this should happen is not impossible, when a general knowledge of the proper motions of all the stars of the 1st, 2d, and 3d magnitudes can be obtained; but then the method of calculation that has been traced out in this and the former Paper, is so perfectly applicable to any new lights observation may throw upon the subject, that a more precise and unobjectionable solar motion can be ascertained by it with great facility. Hitherto we find that a mean rank agrees sufficiently with the phenomena that were to be explained: the apparent velocities of Arcturus and Aldebaran, without a solar motion for instance, were to each other, in the IXth Table, as 208 to 12; our present solar motion has shown, that when the deception arising from its parallactic effect is removed by calculation, these velocities are to each other only as 179 to 85, or as 2 to 1. And though Arcturus still remains a star that moves with great velocity, yet in the XIth Table we have 4 or 5 stars with nearly as much motion; and 4 with more.

Our solar motion also removes the deception by which the motion of a star of the consequence of α Orionis is so concealed as hardly to show any velocity; whereas by computation we find that it really moves at a rate which is fully equal to the motion of the sun.

I must now observe, that the result of calculations founded upon facts, such as we must admit the proper motions of the stars to be, should give us some useful information, either to satisfy the inquisitive mind, or to lead us on to new discoveries. The establishment of the solar motion answers both these ends. We have already seen that it resolves many difficulties relating to the proper motions of the stars, and reconciles apparent contradictions; but our inquiries should not terminate here. We are now in the possession of many concealed motions, and to bring them still more to light, and to add new ones by future observations should become the constant aim of every astronomer.

This leads me to a subject, which though not new in itself, will henceforth assume a new and promising aspect. An elegant outline of it has long ago been laid before the public in a most valuable paper on general Gravitation, under the form of "Thoughts" on the subject;* but I believe, from what has been said in this Paper, it will now be found that we are within the reach of a link of the chain which connects the principles of the solar and sidereal motions with those that are the cause of orbitual ones.

A discovery of so many hitherto concealed motions, presents us with an interesting view of the construction of that part of the heavens which is immediately around us. The similarity of the directions of the sidereal motions is a strong indication that the stars, having such motions, as well as the sun, are acted upon by some connecting cause, which can only be attraction; and as it has been proved that attraction will not explain the observed phenomena without the existence of projectile motions, it must be allowed to be a necessary inference, that the motions of the stars we have examined are governed by the same two ruling principles which regulate the orbitual motions of the bodies of the solar system. It will also be admitted that we may justly invert the inference, and from the operation of these causes in our system, conclude that their influence upon the sidereal motions will tend to produce a similar effect; by which means the probable motion of the sun, and of the stars in orbits, becomes a subject that may receive the assistance of arguments supported by observation.

What has been said in a paragraph of a former Paper, where the sun is placed among the insulated stars,† does not contradict the present idea of its making one of a very extensive system. On the contrary, a connection of this na-

* See the note Phil. Trans. for 1783, page 283. † Ibid. for 1802, page 478.

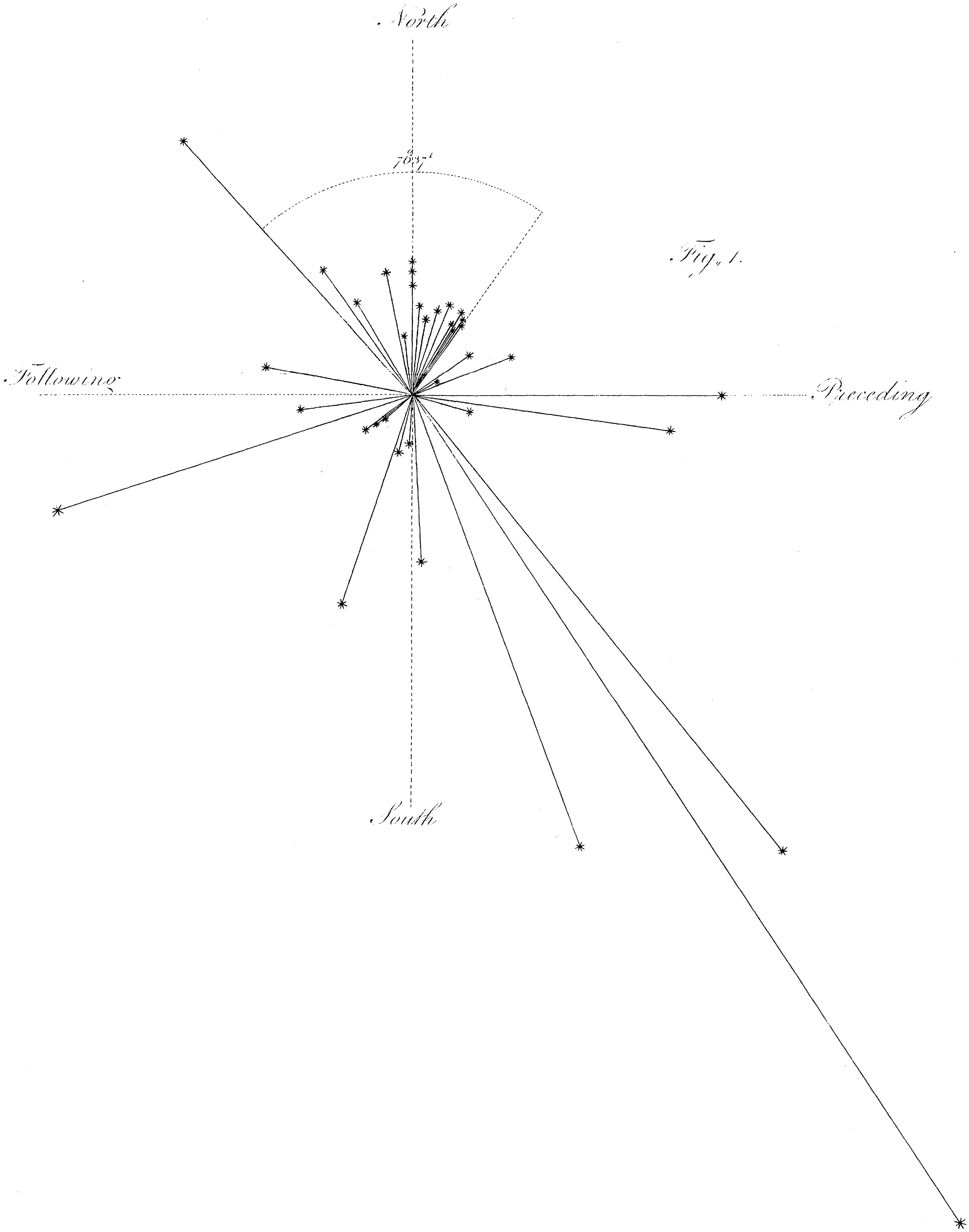

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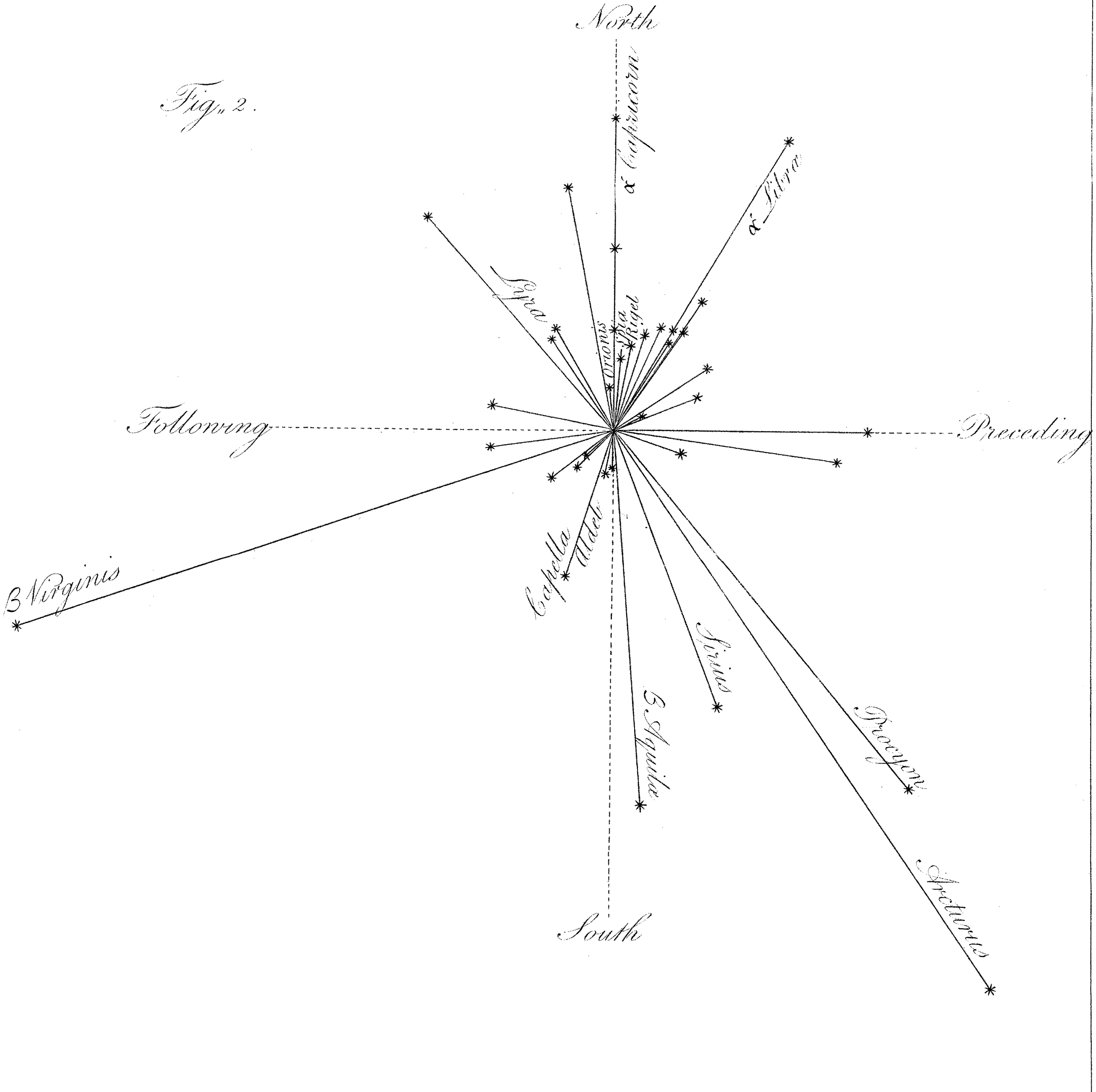
Fig. 2.



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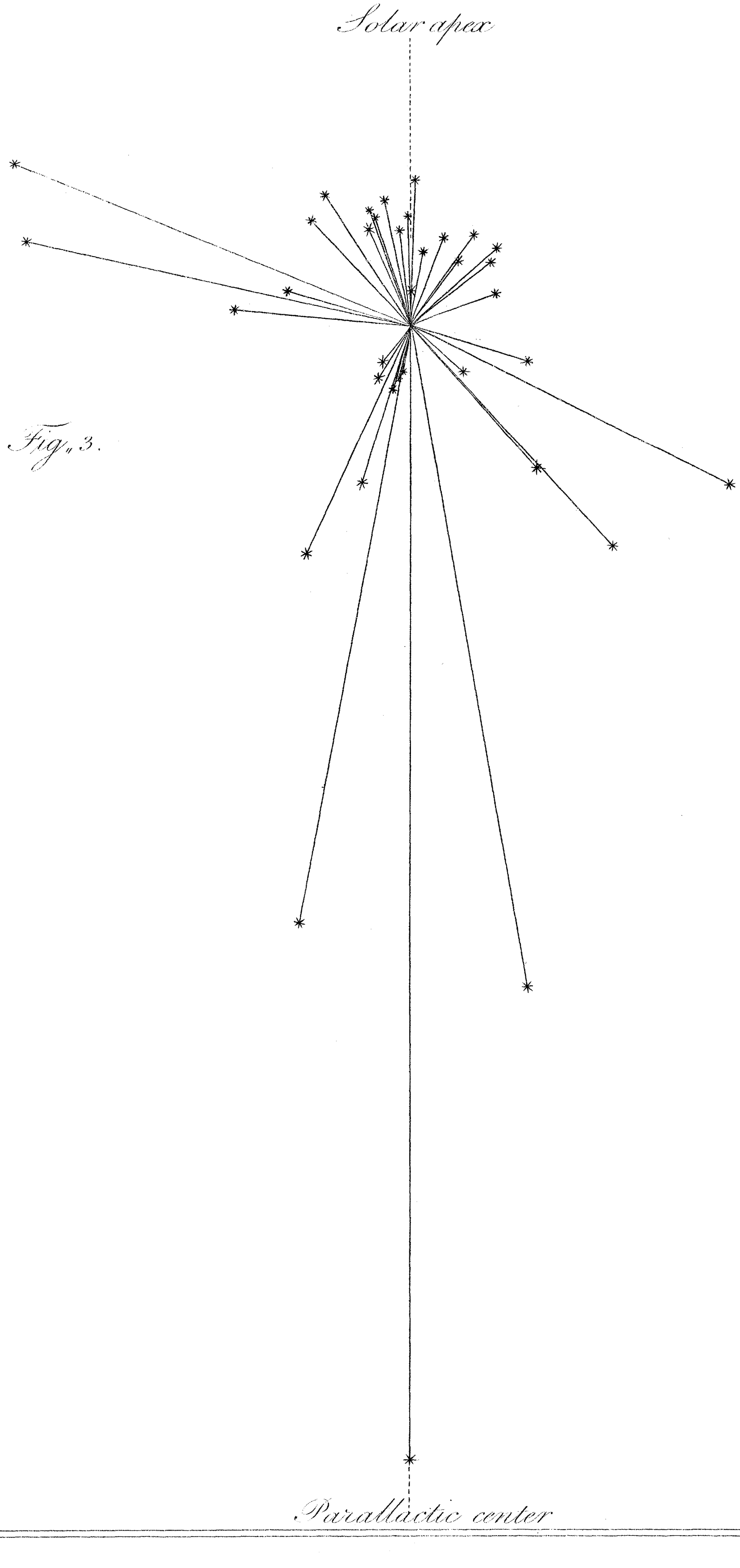

Fig. 3.

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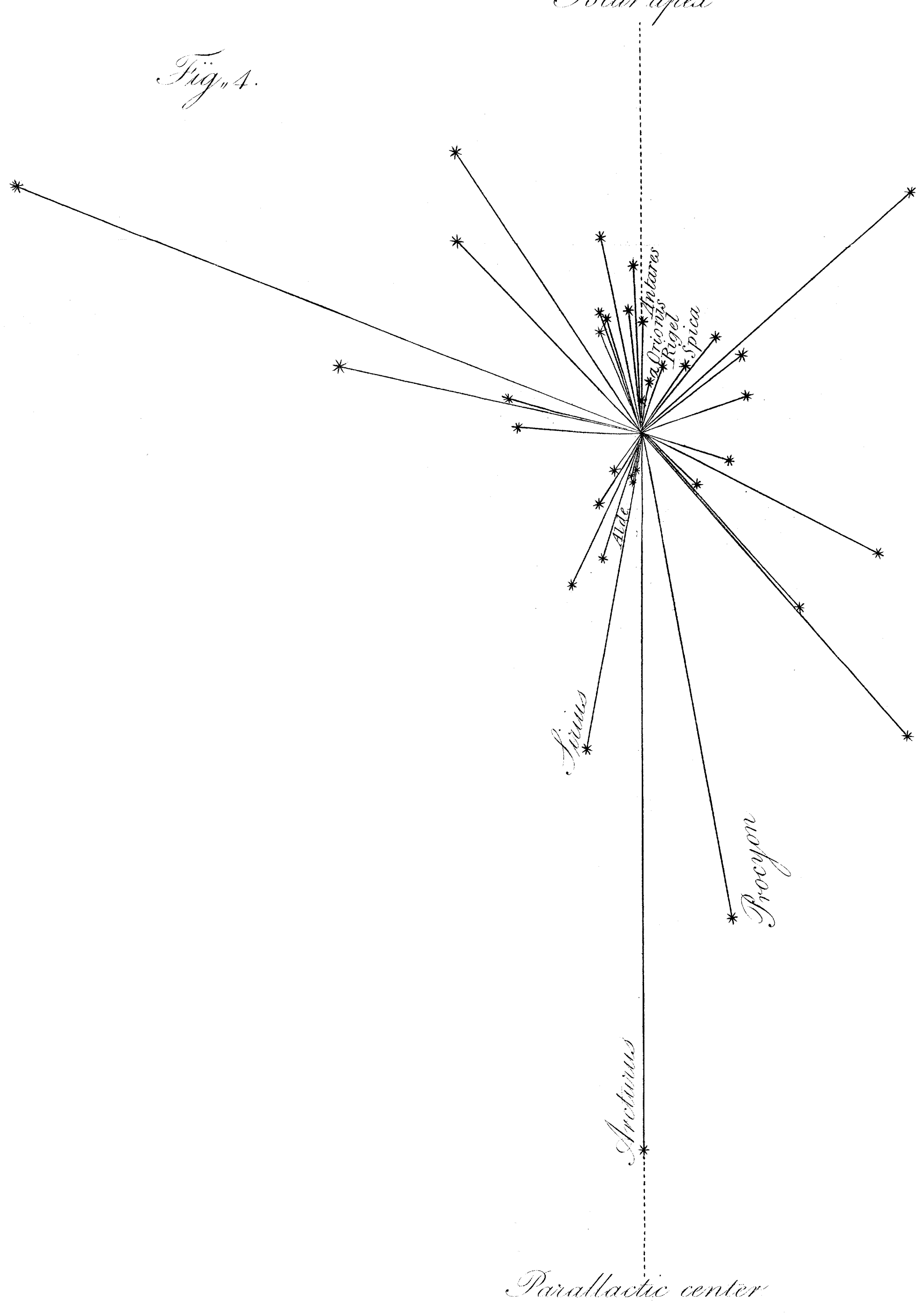
Fig. 4.



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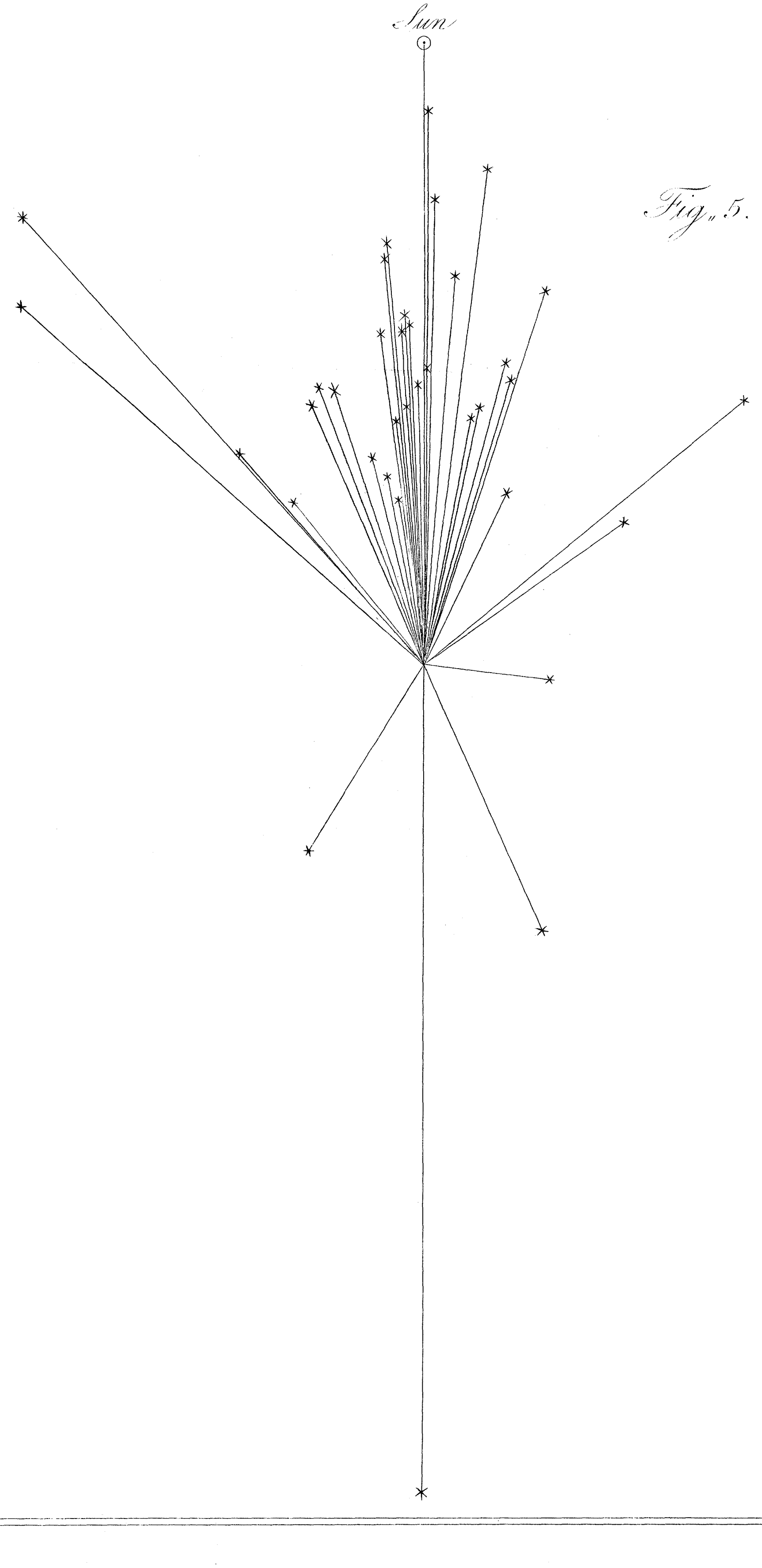

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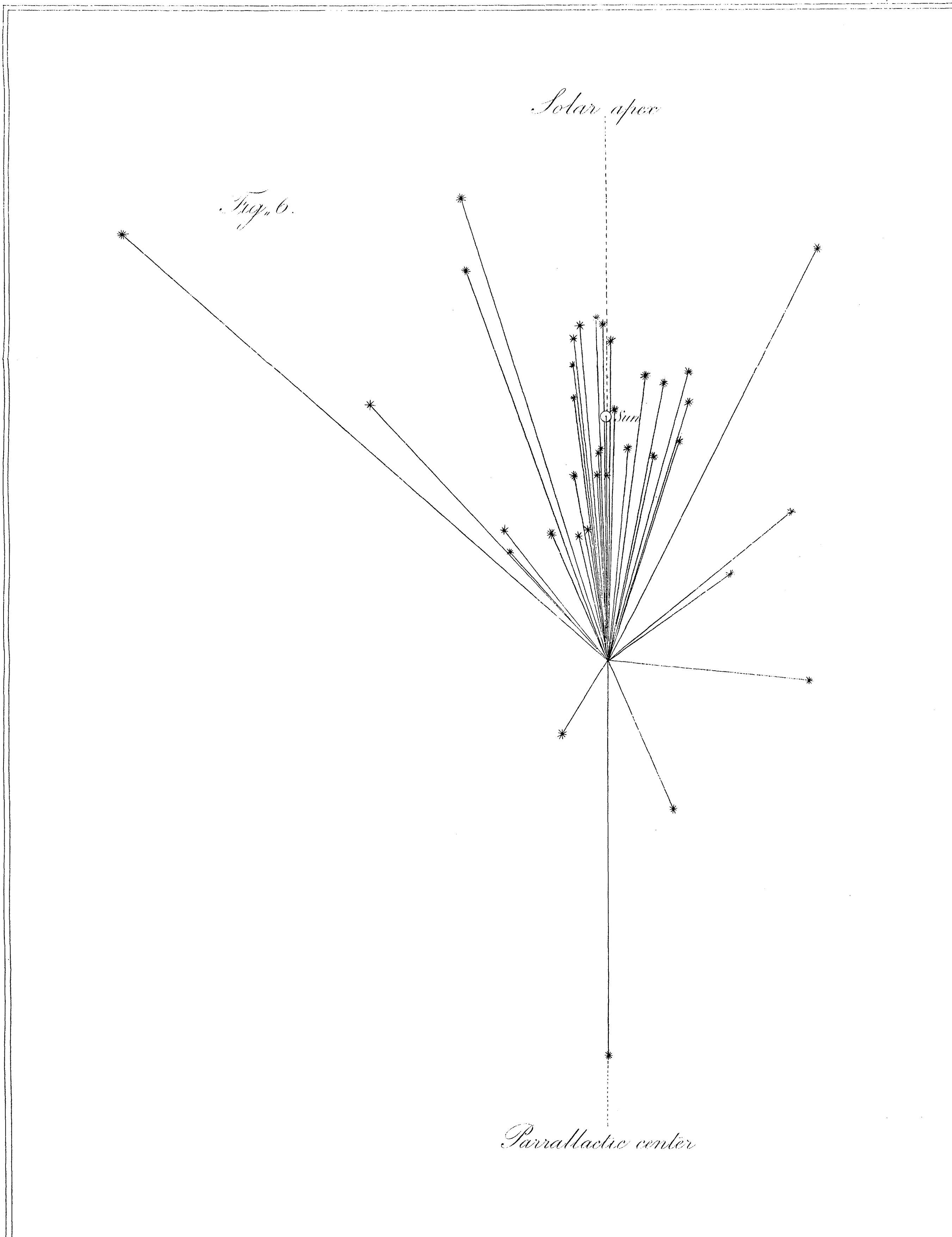

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ture has been alluded to in the same Paper.* The insulation ascribed to the sun relates merely to a supposed binary combination with some neighbouring star; and it has now been proved by an example of Arcturus, that the solar motion cannot be occasioned or accounted for by a periodical revolution of the sun and this or any other star about their common centre of gravity.

* Phil. Trans. for 1802, page 479.

ERRATA

In Table VII. of the first part of this Paper, star Aldebaran, the two last columns,

for 13° 18′ 58″, *read* 13° 41′ 48″.

for 0,02842, *read* 0,02922.

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